Horizontal Transport and Absorption of Solar Radiation in Inhomogeneous Clouds

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The equation of radiative energy balance in homogeneous plane-parallel cloud accounts for the photon transport in vertical direction alone. From this equation, we can evaluate absorption as a difference of the net radiative fluxes measured on the cloud top and bottom. Real clouds exhibit extreme horizontal variability of their optical and, hence, radiative properties. So, the radiative energy balance equation for inhomogeneous clouds includes extra term to describe the net energy transport in the horizontal directions.

In this paper, we use a realistic radiative transfer model for inhomogeneous stratocumulus clouds to study the net energy transport in the horizontal direction. We show that the neglect of this energy is a major source of uncertainty in absorption estimates. Two possible ways of improving the inhomogeneous cloud absorption estimates from field measurements are discussed.

Equation of Radiative Energy Balance

To facilitate discussion, the reflection from surface, as well as scattering and absorption by aerosols and atmospheric gases, will be neglected. Let the clouds occupy the layer $h \le z \le H$ in the Cartesian coordinate system OXYZ. Parallel solar flux F_0 is incident on the cloud top (plane z = H). Consider a pixel bounded by the cloud upper and lower boundaries and the planes x = const, x + Δx = const, and y = const, y + Δy = const. A consequence of the 3-D radiative transfer equation is the law of radiative energy conservation.

$$R(x,y) + T(x,y) + A(x,y) = 1 - E(x,y), \qquad (1)$$

where R(x,y), T(x,y), and A(x,y) are the albedo, transmittance, and absorptance, respectively, while E(x,y) is the ratio of the net radiation flux, lost (E(x,y) > 0) or gained (E(x,y) < 0) through the pixel sides, to the incoming flux $F_0\Delta x\Delta y$. For convenience, we will *term* E(x,y) *the horizontal transport*. Due to (1), the amount of radiative energy reflected, transmitted, and absorbed by a pixel may be either greater or less than 1, depending on the sign of E(x,y).

We let L denote the mean horizontal photon path length in clouds. Major contributors to E(x,y) are those sections of pixel located near its sides and having lengths on the order of L. As pixel size grows, $F_0\Delta x\Delta y$ increases linearly in each of Δx and Δy, whereas the horizontal transport enhances for $\Delta x, \Delta y \le$ L and is almost unchanged when Δx, Δy > L. For this reason, at Δx, Δy >> L, E(x,y) << 1 and equation (1) thus becomes

$$R(x,y) + T(x,y) + A(x,y) = 1 \qquad (2)$$

Given Δx, Δy~L, averaging (1) over such number of pixels N_xN_y, that $N_x\Delta x$ >> L, $N_y\Delta y$ >> L gives

$$\langle E\rangle = \frac{1}{N_xN_y}\sum_{k=i}^{i+N_x}\sum_{m=j}^{j+N_y} E(x_k,y_m)\approx 0,$$

so that equation of the type of (2) is again valid

$$\langle R\rangle + \langle T\rangle + \langle A\rangle = 1 \qquad (3)$$

Here and below, $\langle\cdot\rangle$ denotes an average over realization. It is seen that the realization averaging is equivalent to the pixel stretching.

To calculate the radiative transfer in inhomogeneous clouds, one can use the independent pixel approximation (IPA) (Cahalan 1989; Cahalan et al. 1994a). The IPA neglects the horizontal photon transport, that is, for *any* pixel, it assumes E(x,y)~≡~0 and always uses the energy balance equation (2). Below we show that the neglect of horizontal transport results in uncontrolled errors when determining cloud absorption from field measurements.

Cloud Model and Method of Solution

For more efficient radiative transfer computation, we used the modified fractal model of marine stratocumulus clouds (Cahalan and Snider 1989; Cahalan et al. 1994a,b; Marshak et al. 1994). Model input parameters are the mean $\langle\tau\rangle$, variance D_τ, and the exponent β of the power-law energy spectrum of optical depth modeled as a random process with one-dimensional lognormal distribution and power-law spectrum. A continuous realization of this process is divided into $N_x = 2^{nx}$ pixels of the same horizontal extent $\Delta x = 0.05$ km. Each pixel is assigned τ_i, i = 1,..., N_x, as a value of the random process at the point corresponding to the left-hand side of the pixel, and then the pixel extinction coefficient is calculated as $\sigma_i = \tau_i/\Delta H$, where $\Delta H = H - h$ is the cloud layer thickness. In calculations we used $\langle\tau\rangle = 13$, $D_\tau = 29$, $\beta = 5/3$, and $\Delta H = 0.3$ km, which are typical for marine Sc (Cahalan et al. 1994a).

Numerical simulation of interaction of solar radiation with inhomogeneous stratocumulus clouds is performed with the scattering phase function of Cl cloud (Deirmendjian 1971) calculated from Mie theory for a wavelength of 0.69 µm and single scattering albedo of $\omega_0 = 1.0$. The number of pixels is $N_x = 2^{12} = 4096$ and the length of the cloud realization is 204.8 km. The radiative transfer equation was solved by Monte Carlo method using periodic boundary conditions. Solar incidence is defined by zenith $\xi_\oplus$ and azimuthal $\varphi_\oplus$ angles. The latter is measured from OX - axis and set to zero throughout the computation. For each pixel we calculated albedo, absorptance, and transmittance at the surface level. The mean relative computation error did not exceed 0.6-0.7% while the maximum error was within 1.0 to 1.5%.

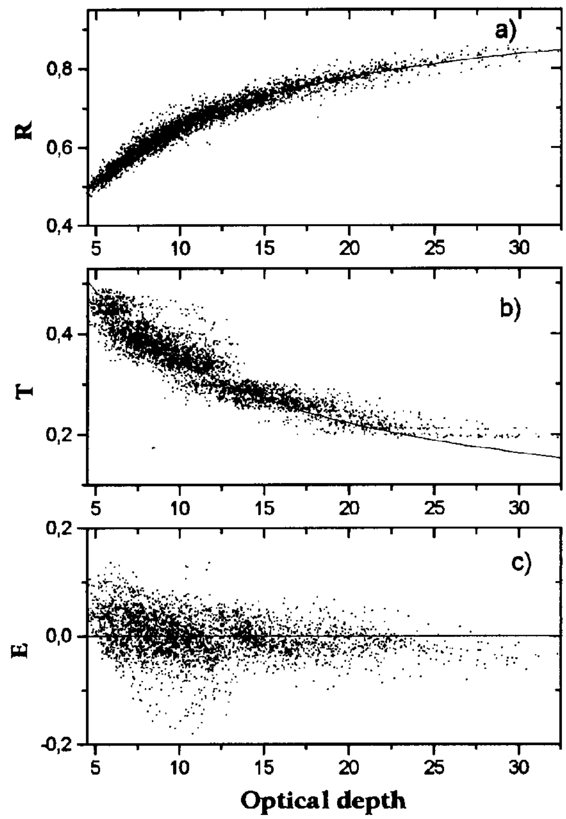

Figure 1. Albedo (a), transmittance (b), and horizontal transport (c) a functions of pixel optical depth with $\xi_\oplus = 60°$ and $A_s = 0$ (ocean). Solid lines show IPA calculations.

Horizontal Radiative Transport

Both in the plane-parallel model and the IPA, radiative properties of clouds are uniquely determined by their optical parameters. Quite the contrary situation occurs for inhomogeneous clouds when the pixels size is less than or nearly equal to L. Because of the horizontal nonuniformity of radiative fluxes, two pixels with the same optical thickness but different neighbors' optical properties may appear to have different A, T, and E. Typically, $|E|$ is of about the same order of magnitude as the other terms in the energy balance equation (Figure 1), and, therefore, all of the methods for interpreting field data based on the energy balance equation should, as possible, take the horizontal photon transport into account.

Pixels with $\tau_i < 5$ have horizontal optical depths $\tau_{i,x}$ of less than 1. Most photons traverse them without scatter, so they predominantly loose ($E(x_i) > 0$) radiation through their sides (Figure 1). The reverse is true for optically thick pixels, with $\tau_i > 25$ and $\tau_{i,x} > 5$. The larger the pixel optical depth, the smaller is the region located near pixel sides and playing major part in the radiation interaction of pixels; hence the less is $|E|$.

Figure 2 presents numerical realizations of random functions $\tau(x)$ and $E(x)$, obtained with fixed parameters of one-point lognormal distribution and different values of exponent β characterizing the slope of the power-law energy spectrum of optical depth. Because of the large cloud optical depth, radiation leaking out the pixels sides interacts with nearby pixels only, and cannot interact with pixels ~10 km or more apart. This explains the insensitivity of $E(x)$ to the macroscale fluctuations of $\tau(x)$. Further, at $\beta = 2.9$, $\tau(x)$ is a smooth function, while the neighboring pixels are of approximately the same optical depth. For each pixel, the loss and gain of radiative energy through pixel sides nearly compensate each other, $E(x) \approx 0$, so that simpler energy balance equation (2) can be used. Thus, the slope of the energy spectrum (or the fractal dimension) of cloud optical depth represents one of the fundamental parameters governing the horizontal photon transport in inhomogeneous clouds.

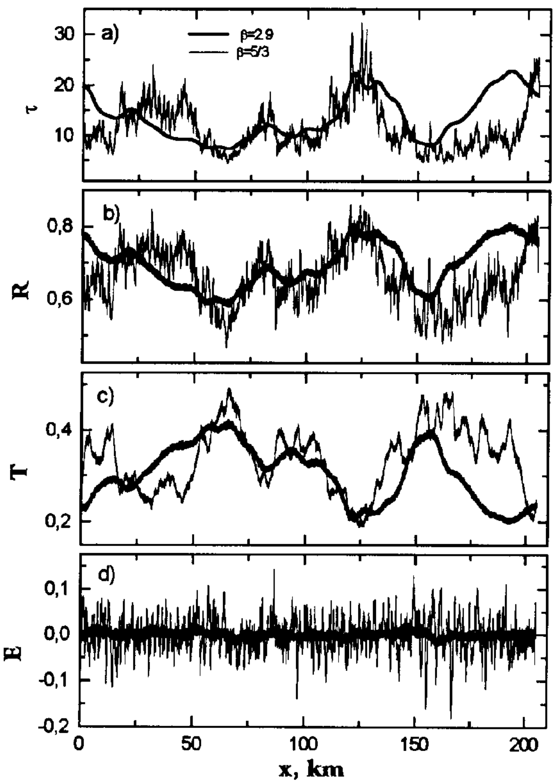

Figure 2. Numerical realizations of optical depth (a), albedo (b), transmittance (c), and horizontal transport (d) with $\xi_{\oplus} = 60^0$, $A_s = 0$ (ocean) and for different slopes β of power-law energy spectrum of optical depth.

Cloud Absorption

According to formula (1), absorption of solar radiation by some cloud volume can be determined, provided the net fluxes leaving through its closed surface are known. In practice, however, only net fluxes out cloud top and bottom, R(x,y) and T(x,y), are measured. Then, equation (1) fails to predict actual absorption A(x,y). When E(x,y) is neglected and the balance equation (2) is used, an inferred absorption, A′(x,y), not the real one, A(x,y), is determined according to formula A′(x,y) = A(x,y) + +E(x,y) == 1 - R(x,y) - T(x,y). In such a case, E(x,y) is interpreted as some apparent absorption. Since E(x,y) may be either positive or negative, and be of same order as A(x,y), the inferred absorption A′(x,y) may substantially diverge from the real one.

To estimate the E(x,y) effect on the accuracy of determining absorption from field measurement, we will use the one-dimensional cloud model with $\beta = 5/3$. For convenience and better understanding of physics, in the IR range we will account for the water droplet absorption alone and assume the single scattering albedo to be $\omega_{0,ir} = 0.99$.

The absorptance $A_{ir}(x_i)$ is shown in Figure 3 in terms of the function $A^{/}_{ir}(x_i) = A_{ir}(x_i) + E_{ir}(x_i)$, $i = 1,\cdots,4096$. It is seen that $A_{ir}(x_i)$ is extremely sensitive to the optical depth of a given and neighboring pixels, and may vary by almost a factor of four. $A_{ir}(x_i)$ and $A^{/}_{ir}(x_i)$ are not uniquely related, and, say, $A^{/}_{ir}(x_i) = 0.20$ suggests the real absorptance to be in the range $0.08 \le A_{ir}(x_i) \le 0.32$. Thus, it is *impossible* to estimate true absorption using *individual* net flux measurements on the top and bottom of the inhomogeneous clouds. Very few cases are conceivable when, due to the horizontal transport, a pixel receives more radiation than it absorbs, and then $A^{/}_{ir}(x_i) < 0$. This may be the reason for the occasional inference of negative absorption values from field measurements (Rawlins, 1989; Stephens and Tsay, 1990).

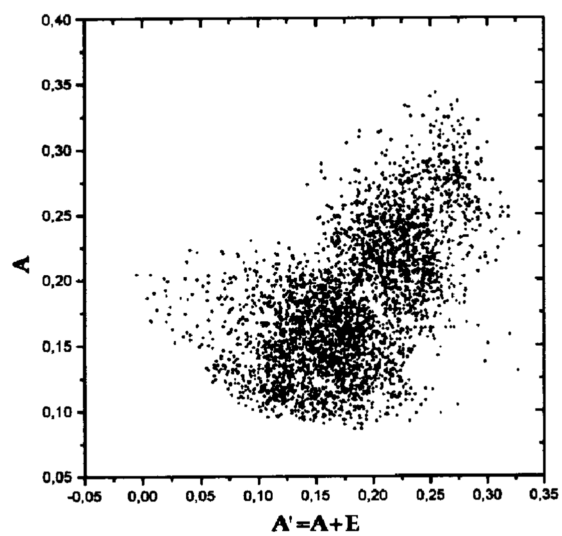

Figure 3. Absorptance A as a function of inferred absorptance A′ = A + E with $\xi_{\oplus} = 60^0$, $A_s = 0$ and $\omega_{0,ir} = 0.99$.

We suggest two possible approaches to improve the absorption estimates.

1. After the net fluxes are spatially averaged, $\langle E \rangle = 0$, and the mean absorptance $\langle A \rangle$ is uniquely determined by formula (3). Before using formula (3), the question in order is: *What is the minimum averaging area?*

Let us average the horizontal transport over different numbers of pixels $E(nx) = \frac{1}{2^{nx}}\sum_{i=1}^{2^{nx}} E(x_i)$, $nx = 0,1,\cdots$. Analogous formulas can be written for albedo, transmittance, and absorptance. The length l(nx) of the averaging segment of realization is $l(nx) = \Delta x \cdot 2^{nx}$. The effect of clouds on solar absorption is commonly quantified in terms of the ratio r(nx) of cloud radiative forcing at the surface level to that at the top of the atmosphere. Under the assumptions above, this ratio is

$$r(nx) = \frac{1-T(nx)}{R(nx)} = 1 + \frac{A(nx) + E(nx)}{R(nx)} \qquad (4)$$

From (4) it follows that, for arbitrary nx, r(nx) depends on both absorption and horizontal transport. The ratio r(nx) will *uniquely* determine the cloud absorption only when $nx > (nx_*)$, where nx_* is given by the inequality $A(nx_*)$.

The horizontal transport, E(nx), and r(nx) were calculated for a range of single scattering albedo and are shown in Figure 4. At $nx > nx_* \approx 2^7$, the horizontal transport is negligible, so the ratio r(nx) nearly levels off, truly characterizing cloud absorption. Thus, the net flux measurements of solar radiation, taken above and below stratocumulus clouds, do can provide reliable estimates of absorption, if the fluxes are averaged over the realization's fragments $l(2^7)$~6.0 km or longer.

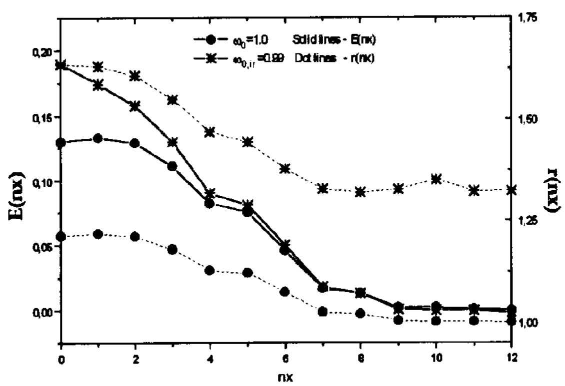

Figure 4. Dependence of the horizontal transport and the ratio of cloud radiative forcings on the number of pixels over which the average is taken: $\xi_\oplus = 60^0$ and $A_s = 0$.

2. Suppose that the data are available from albedo and transmission measurements in the visible (subscript "vis") and near IR (subscript "ir") spectral range. In the visible range, the absorptance $A_{vis}(x,y) = 0$, and so one finds $E_{vis}(x,y) = 1 - R_{vis}(x,y) - T_{vis}(x,y)$. Now let us assume that the function $E_{ir}(x,y) = f\big(E_{vis}(x,y)\big)$ is known; then, for absorptance in the infrared we have

$$A_{ir}(x,y) = 1 - R_{ir}(x,y) - T_{ir}(x,y) - f\big(E_{vis}(x,y)\big) \qquad (5)$$

The function $E_{fis}(x,y) = f\big(E_{vis}(x,y)\big)$ can be found by mathematical simulation. Using a linear regression of E_{ir} versus E_{vis}, we get $E_{ir} = 1.2E_{vis}$ (Figure 5a). Substituting this result in (5) yields an improved estimate of the absorption A'. From comparison of results in Figures 3 and 5 we see that the use of simple "measurement" scheme considered here allows significant improvement of the cloud absorption estimate. This conclusion is also valid with different values of surface albedo and solar zenith angle.

The results of Figures 4 and 5 are paradoxical, at the first sight: the horizontal photon transport with absorption, $|E_{ir}|$, may be greater than without it, $|E_{vis}|$. Since $\langle E_{ir}\rangle = \langle E_{vis}\rangle = 0$, the distribution of E_{ir} is broader than that of E_{vis}. his effect can be explained by the following argument. Consider a segment of photon trajectory between nth and n+2nd collisions. Let nth and n+2nd collisions belong to the pixel with number i, while n+1st collision to either of the i+1st and i-1st pixels. In other words, the photon exits the pixel, suffers a collision in the neighboring one, and then returns back. In the pure scattering case, the photon statistical weight, proportional to its radiative energy, does not change upon collision, thus, such trajectory segment contributes nothing to the horizontal transport. In the presence of absorption by water droplets, however, the photon leaves the pixel having one statistical weight, and returns with other, less one; thus, the horizontal transport in nonzero. This means that, switching to the absorptive case, the number of trajectories contributing to E_{ir} increases and, hence, the E_{ir} distribution broadens. If the argument above is valid, the addition of the atmospheric gaseous absorption should further broaden the distribution of E_{ir}.

Conclusion

We have found that the radiative effects of inhomogeneous clouds allow us to explain the cloud absorption anomaly. In typical stratocumulus clouds, the horizontal photon transport, being zero in the plane-parallel model, is comparable (in the order of magnitude) to the other terms in the energy balance equation. When absorption is determined as a difference between the net radiative fluxes measured above and below the clouds, the horizontal transport is interpreted as apparent absorption and is a major source of uncertainty.

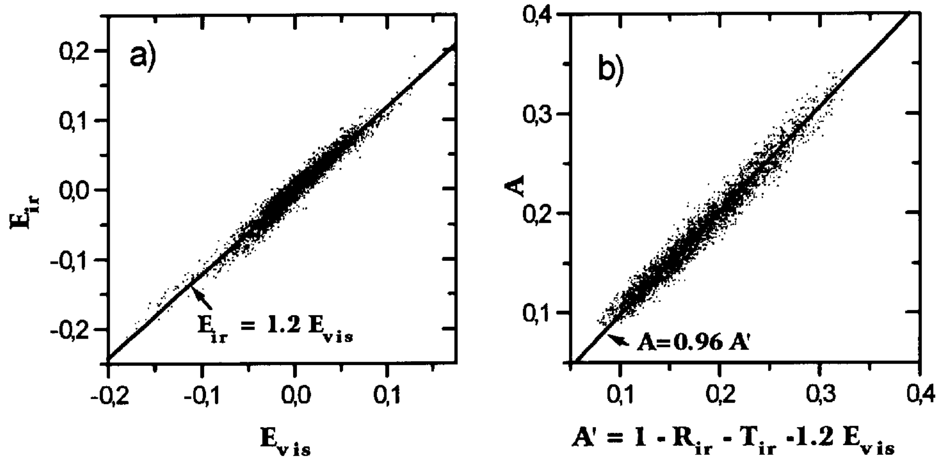

Figure 5. Linear regression of E_{ir} versus E_{vis} (a), and absorptance as a function of improved absorptance estimate A'(b) with $\xi_{\oplus} = 60^0$ and $A_s = 0$ (ocean).

Using space averaging or simultaneous measurements of visible and shortwave radiation, it is possible to substantially improve the estimate of absorption by inhomogeneous clouds.

We plan to verify our findings, using for mathematical simulation the data of field measurements, primarily ARESE data.

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